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The zero mass limit in Yang–Mills theory II

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Abstract. The zero mass limit of renormalizable theories involving massive Yang–Mills fields is investigated. These theories can be classified in this limit in terms of the charge of the zero mass scalar fields introduced in the previous paper.

1. Renormalizable gauge theories

In the previous paper (Dombey 1976 to be referred to as I) one of us considered the zero mass limit for the scattering of a massive Yang–Mills field off a charged scalar source. In this paper we shall show that the results of paper I persist in a renormalizable gauge theory involving a triplet of massive Yang–Mills fields.

We shall continue to take the simple viewpoint of paper I by initially considering the elastic scattering of two charged massive photons of mass m, $\gamma^+\gamma^- \to \gamma^+\gamma^-$. An unpublished note by J C Taylor (1972)† showed that the problems of renormalizability and tree-unitarity manifest themselves in lowest order in this process. The diagrams of figure 1 give the amplitude for the scattering of longitudinally polarized photons in the centre of mass frame

$$T_{\text{LLLL}} = \tfrac{1}{2}e^2(p/m)^2(1+\cos\theta) \tag{1}$$

as $p \to \infty$ for fixed scattering angle θ. This behaviour shows that without modification the theory is unrenormalizable and the $m \to 0$ limit does not exist. Incorporating the scalar fields ϕ of paper I into the theory cannot by itself change this behaviour.

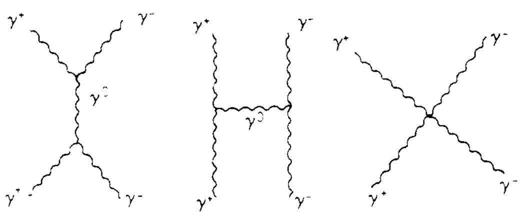

Figure 1.

Incorporating a neutral scalar particle ψ (figure 2) with a coupling to the $\gamma^+\gamma^-$ system proportional to em as given in spontaneously broken gauge theories removes this singular behaviour. Now

$$T_{\text{LLLL}} = (p/m)^2(1+\cos\theta)[A_{\gamma 0} + A_{\psi}] \tag{2}$$

† Entitled: *The Physical Role of Scalar Particles in Convergent Theories of Charged Vector Particles, Oxford.*

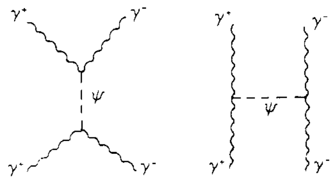

Figure 2.

as $p \to \infty$ where

$$A_\psi = -A_{\gamma 0} = \tfrac{1}{2}e^2. \tag{3}$$

This essentially is Taylor's result, although his calculation was performed for the Weinberg model.

Our interest, however, is in the zero mass limit of T_{LLLL}. So as $(p/m) \to \infty$, we need to consider the next term in the expansion in powers of (p/m); i.e. the constant T^0 in

$$T_{\text{LLLL}} = (p/m)^2(1+\cos\theta)[A_{\gamma 0} + A_\psi] + T^0 + \mathrm{O}((m/p)^2). \tag{4}$$

A straightforward calculation gives

$$T^0 = (\tfrac{1}{2}e)^2\left(\cos\theta - \frac{3+\cos\theta}{1-\cos\theta} + 2\lim_{m\to 0}\frac{\mu^2}{m^2}\right) \tag{5}$$

where μ is the mass of the scalar ψ.

This result must be interpreted in the same way as the result of paper I in terms of a triplet of zero mass scalar fields $\boldsymbol{\phi}$ which define the longitudinal modes of the Yang–Mills field in the zero mass limit. That is to say, we have as before a Lagrangian describing the underlying zero mass theory

$$\mathscr{L} = -\tfrac{1}{4}\boldsymbol{G}_{\mu\nu}\cdot\boldsymbol{G}_{\mu\nu} - \tfrac{1}{2}(\partial_\mu\boldsymbol{\phi} + f\boldsymbol{\phi}\times\boldsymbol{A}_\mu)^2 + h(\boldsymbol{\phi}^2)^2 + \bar{\mathscr{L}}(\psi) \tag{6}$$

where $\mathscr{L}(\psi)$ contains the ψ field which only contributes to higher orders. Fadeev–Popov ghosts will also be needed in the Lagrangian (6) for the calculation of higher orders. As before f determines the coupling of the $\boldsymbol{\phi}$ current to $\boldsymbol{A}_\mu$.

The result of equation (5) is then given by the diagrams of figure 3

$$T(\phi^+\phi^- \to \phi^+\phi^-) = f^2\left(\cos\theta - \frac{3+\cos\theta}{1-\cos\theta}\right) + 4h \tag{7}$$

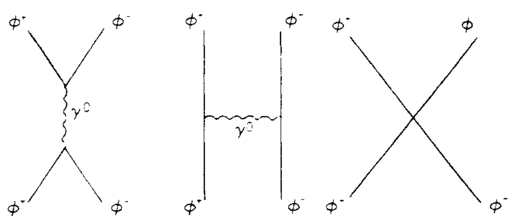

Figure 3.

So just as in the simplified example of paper I

$$f = \tfrac{1}{2}e. \tag{8}$$

The coefficient of the term with the pole at $\cos\theta = 1$ gives a definition of charge as particles of charge $\pm Q$ will interact via one-photon exchange at large distances with the amplitude

$$-4Q^2/(1-\cos\theta). \tag{9}$$

So we see that the charge of the $\phi^{\pm}$ is $\frac{1}{2}e$.

Thus the result of paper I that the underlying zero mass Lagrangian of a massive Yang–Mills theory must contain the scalar fields $\boldsymbol{\phi}$, which however have fractional charge, is still true in a renormalizable gauge theory (for example, that given by 't Hooft 1971) which contains a triplet of Yang–Mills fields of mass m. The conventional Yang–Mills theory involving zero mass particles with two allowed polarization states is not the underlying theory.

Finally comparing equations (5) and (7) we have the result

$$\lim_{m\to 0}(\mu^2/m^2) = 8h/e^2 \tag{10}$$

which is the case in the 't Hooft theory.

2. Higgs theories

The previous section shows that although there exists a renormalizable theory of massive Yang–Mills fields, nevertheless the inherent conflict between Lorentz invariance and the non-Abelian gauge invariance still manifests itself in the zero mass limit in the breakdown of local gauge invariance and charge universality.

This, however, is not the only possibility. The simplest procedure is to directly break the isotopic symmetry; this is the method originally used by Higgs (1964). To do this here, we require that the Higgs scalar ψ is just ϕ^0. (That is to say, take the triplet $\boldsymbol{\phi}$ and associate a non-zero vacuum expectation value to the neutral component ϕ^0; then there is a physical field ϕ^0_{ph} where $\phi^0 = \bar{\phi}^0 + \phi^0_{\text{ph}}$, $\langle\phi^0\rangle = \langle\boldsymbol{\phi}^0\rangle \neq 0$, $\langle\phi^0_{\text{ph}}\rangle = 0$. We now refer to ϕ^0_{ph} as ϕ^0.) Mass is thereby induced in the charged components $\gamma^{\pm}$ which we shall now call $W^{\pm}$ leaving the photon γ^0 massless, thus breaking the isospin symmetry globally. These are the vector mesons of the Georgi–Glashow model (Georgi and Glashow 1972); the theory is also described by 't Hooft (1971).

Consider the process $W^+W^- \to W^+W^-$. As in equations (2) and (3) it is still true that $A_{\gamma 0} + A_{\psi} = 0$. This is sufficient for a tree-unitary theory, but we need to recalculate T^0 of equation (4). γ^0 exchange (for example in the s channel) involved a pole at $s = m^2$ when the γ^0 had mass m, but now the pole is at $s = 0$. So whereas there was a term proportional to

$$\frac{1}{s-m^2} = \frac{1}{4p^2+3m^2} = \frac{1}{4p^2}\left(1 - \frac{3}{4}\frac{m^2}{p^2}\right) \tag{10a}$$

there now is a term proportional to

$$\frac{1}{s} = \frac{1}{4p^2+4m^2} = \frac{1}{4p^2}\left(1 - \frac{m^2}{p^2}\right). \tag{10b}$$

Hence there now is a contribution to $A_{\gamma 0}$ of equation (4) which differs by a term proportional to (m^2/p^2) from the previous case. A similar calculation can be performed for the t channel γ^0 pole. T^0 is then no longer given by equation (5); instead it can easily be calculated to be

$$T^0 = e^2\left(\cos\theta - \frac{3+\cos\theta}{1-\cos\theta} + 2\lim_{m\to 0}\frac{\mu^2}{m^2}\right). \tag{11}$$

Thus the zero mass limit of the broken symmetry or Higgs theory we have described is given by the underlying Lagrangian

$$\mathscr{L} = -\tfrac{1}{4}\boldsymbol{G}_{\mu\nu}\cdot\boldsymbol{G}_{\mu\nu} - \tfrac{1}{2}(\partial_\mu\boldsymbol{\phi} + e\boldsymbol{\phi}\times\boldsymbol{A}_\mu)^2 + h(\boldsymbol{\phi}^2)^2. \tag{12}$$

So the zero mass limit of a Higgs theory involves scalar particles $\phi^\pm$ of integral charge e. From this point of view, the importance of breaking the symmetry directly in this way is that it allows charge universality to be still satisfied in the zero mass limit.

3. Gauge transformations

We now go back to the discussion at the end of paper I and consider the gauge transformations of the two theories described in the zero mass limit. The conclusion of paper I was that some form of gauge symmetry breaking was necessary in non-Abelian gauge theories involving massive vector mesons, as otherwise the zero mass limit itself breaks the gauge symmetry.

This we see is precisely what happens in the two types of theory under consideration here. In the 't Hooft model considered in § 1, the Lagrangian of equation (6) is so constructed as to be invariant under the transformation ('t Hooft and Veltman 1972)

$$\begin{aligned} &\boldsymbol{A}_\mu \to \boldsymbol{A}_\mu + \partial_\mu\boldsymbol{\Lambda} + e\boldsymbol{\Lambda}\times\boldsymbol{A}_\mu \\ &\boldsymbol{\phi} \to \boldsymbol{\phi} + \tfrac{1}{2}e\boldsymbol{\Lambda}\times\boldsymbol{\phi} - \tfrac{1}{2}e\psi\boldsymbol{\Lambda} \\ &\psi \to \psi + \tfrac{1}{2}e\boldsymbol{\Lambda}\cdot\boldsymbol{\phi}. \end{aligned} \tag{13}$$

Thus a new isoscalar particle ψ must be introduced into the theory and the theory is now invariant under a distorted gauge invariance, not normal local isotopic gauge invariance.

The broken symmetry Higgs theory of § 2 however is invariant in the zero mass limit in which the symmetry is restored under the normal isotopic gauge transformation

$$\begin{aligned} &\boldsymbol{A}_\mu \to \boldsymbol{A}_\mu + \partial_\mu\boldsymbol{\Lambda} + e\boldsymbol{\Lambda}\times\boldsymbol{A}_\mu \\ &\boldsymbol{\phi} \to \boldsymbol{\phi} + e\boldsymbol{\Lambda}\times\boldsymbol{\phi}. \end{aligned} \tag{14}$$

So when mass is induced in the two theories, the distorted gauge invariance allows an equal mass globally isospin-invariant Yang–Mills triplet at the price of fractional charges in the underlying theory, whereas normal isotopic invariance and integral charges in the underlying theory are obtained only by directly breaking the isotopic symmetry.

The gauge invariance of the Lagrangian persists in the massive vector meson theories ('t Hooft and Veltman 1972); this is necessary for renormalizability. In both cases a term proportional to $m\Lambda$ must be added to the gauge transformation for $\boldsymbol{\phi}$ in equations (13) and (14). $\phi^\pm$ are then no longer physical particles but just subsidiary gauge fields; the physical longitudinal state of the charged vector meson is the gauge invariant state constructed from $A_\mu^\pm$ and $\phi^\pm$.

4. Discussion

We have seen that there are two kinds of massive Yang–Mills theory which can be classified by their underlying zero mass Lagrangians. This underlying Lagrangian is not the conventional Yang–Mills Lagrangian but includes scalar fields.

It is thus interesting to classify physical symmetry theories according to these criteria. A unified theory of weak and electromagnetic interactions should, presumably, be unified in the zero mass limit. As a zero mass particle actually exists in this theory, this zero mass limit should be physically sensible and hence, the considerations of this paper suggest that the appropriate theory should be one of the Higgs broken symmetry type. The Georgi–Glashow model is of this type but only contains one neutral vector meson. As neutral weak currents exist, a rank two simple group such as SU(3) should thus be considered. The Weinberg model, although a renormalizable theory of weak and electromagnetic interactions, is not a unified theory: the electromagnetic charge is not the weak charge, however these are defined.

On the other hand, in a symmetry theory of strong interactions, a gauge theory of the 't Hooft type, for SU(3) for example, would be appropriate. It is true that fractional charges would appear in the underlying zero mass Lagrangian but there is no compelling reason for considering the zero mass (or high energy) limit of the theory as being anything other than an asymptotic limit because there are no physical zero mass hadrons.

Three important questions remain. First, do the longitudinal modes have any connection in unified theories of weak and electromagnetic interactions with the existence or otherwise of magnetic monopoles? Secondly, do the fractionally charged scalar particles in theories of the 't Hooft type have any relation to fractionally charged quarks in theories of strong interactions? Thirdly, are Fadeev–Popov ghosts necessary for calculations with the zero mass Lagrangians of this paper if the scalar fields ϕ are restricted to external lines? There is now no obvious violation of unitarity.

Finally, another mechanism has been pointed out which can give rise to massive Yang–Mills fields. This is the dynamical symmetry breakdown mechanism of Coleman and Weinberg (1973). Here the basic fields all have zero mass and are therefore fully gauge invariant but the interactions of the theory lead to a dynamical vector meson mass being induced.

The arguments of this paper do not apply to such theories which do not necessarily contain scalar particles. The colour SU(3) Yang–Mills theory of quarks and gluons (Fritzsch *et al* 1973) of strong interactions is an example of a theory where masses which are thought to enter in this way.

Acknowledgments

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Note added in proof. Charge normally defines a long-range static interaction which therefore cannot depend on helicity. The Georgi–Glashow model contains $W^{\pm}$ of mass m and a zero mass photon; hence a $W^{+}W^{-}$ system can be found with both particles moving arbitrarily slowly and interacting, although arbitrarily far from each other. So even in the $m = 0$ limit, the generalized charge defined by (9) does not depend on helicity.

On the other hand in the 't Hooft model, the particles $\gamma^+\gamma^0\gamma^-$ are all of mass m and hence the interaction between $\gamma^+\gamma^-$ is not long range except in the limit $m = 0$ where the interaction cannot be taken as static. So in this case the generalized charge can, and does, depend on helicity.

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